Lateral Equilibrium of Asymmetrical Swept Wings: Aileron Control vs Geometric Twist

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A static aeroelastic phenomenon unique to an aircraft with asymmetrically swept wings is discussed. A simple formula is derived from the analysis of a highly idealized model. The validity of this formula is examined through the use of a more sophisticated numerical analysis. Among the results of this analysis are the following: aileron settings of a few degrees are sufficient to trim such aircraft in roll-for-g flight; the use of built-in twist in the form of initial negative dihedral provides an efficient alternative to aileron trim; if the wing is elastically tailored in a proper fashion, it may be possible to design a wing whose elastic deformation under airloads provides a form of self-trim in roll at the cruise q of the aircraft.

Introduction

THE oblique wing aircraft concept is currently being explored as a possible method of achieving high lift-to-drag (L/D) ratios at high transonic and low supersonic speeds.[1,2] Prominent among the features of this unconventional aircraft is a wing of relatively large unswept aspect ratio (of the order of 8-12) which can be pivoted so that it presents itself at an oblique angle to the flow (Fig. 1). Theoretically, this asymmetrical sweeping of a high aspect ratio wing results in a very efficient wing shape in this speed range.[3] That this theory is valid has been shown in wind tunnel tests at the NASA Ames Research Center.[4] While demonstrably advantageous to the aerodynamicist, such a design suggests potential structural stiffness and strength difficulties which deserve careful consideration. Of these difficulties, the areas of static aeroelasticity and flutter seem most important.

The term static aeroelasticity is commonly applied to aeroelastic problems where inertia effects can be neglected. Control effectiveness and the redistribution of airloads on a flexible aircraft are prime examples of problems where the equilibrium state of the flight vehicle is highly dependent upon the interaction between the airloads and the flight structure. On the other hand, static wing divergence provides an example of a static aeroelastic stability problem. Since this latter problem is of no concern to a freely flying oblique winged aircraft,[2] the attention of this paper will be focused only on the problem of flexible wing airload redistribution and the lateral trim requirements of flexible wings which are asymmetrically swept.

Earlier studies have focused attention of stability and control characteristics which might be of importance to oblique wing aircraft design.[5-7] In Ref. 7, the author very briefly explored the possible influence of static aeroelasticity on oblique wing design. Further work has shown that the analysis presented in that reference can be extended. The present study is divided into three parts: the analysis of the potential aileron control requirements for an oblique winged aircraft to ensure lateral equilibrium at various flight speeds; the presentation, analysis and comparison of an alternative mode of ensuring lateral equilibrium, the use of built-in geometric twist; and, finally, an assessment of the validity of the assumptions used and results obtained in the latter two studies.

Discussion

Some insight into the magnitude and importance of the oblique wing lateral trim problem can be obtained through the study of the simple elastic wing model considered in Ref. 7. The analysis of this model, shown in Fig. 2, will seek to determine the extent to which aileron deflection, or some other means of control, is necessary to ensure lateral equilibrium. Although the analysis of the use of aileron deflection for this same model has been briefly discussed in Ref. 7, the essential features of that analysis will be reviewed here and the results extended because they bear heavily upon the ensuing geometric twist analysis.

The idealization shown in Fig. 2 represents a continuous, uniform-property, elastic wing, of constant chord, swept at an angle Λ to the direction of flight. The wing is uncambered and has full-span ailerons for lateral control. For the formulation of the analytical model, the wing is assumed to behave as a slender beam with a straight elastic axis. Torsional flexibility of the wing is ignored to simplify the analysis. Finally, aerodynamic strip theory is used to describe both the applied loads and the aeroelastic loads. With reference to Fig. 2, the governing differential equation of equilibrium of the flexible wing, under the usual elementary beam theory assumptions can be written (in nondimensional form (Ref. 8,pp. 479-481) as

$$d^4w/d\eta^4 + (\lambda dw/d\eta) = (p_o L^3/EI) + \beta\delta(\eta) \qquad -1 \le \eta \le 1 \quad (1)$$

where

$w(\eta)$ = bending deformation, nondimensionalized with respect to L
p_o = constant load per unit length in the η direction
EI = bending stiffness, a constant
λ = $qcc_{L\alpha}L^3 \sin\Lambda \cos\Lambda/EI$
β = $qcc_{L\delta}L^3 \cos^2\Lambda/EI$
q = dynamic pressure

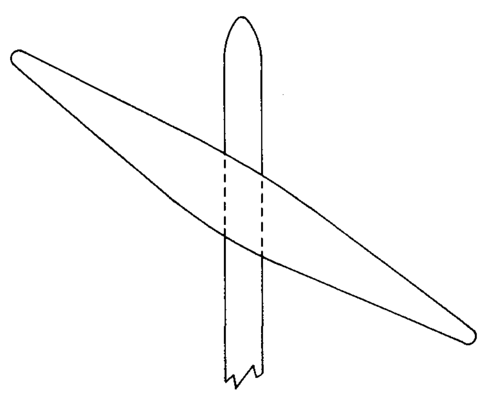

Fig.1 Typical oblique wing planform configuration.

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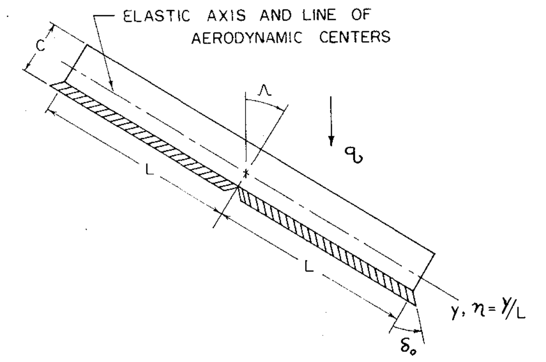

Fig. 2 Idealized oblique wing analysis model.

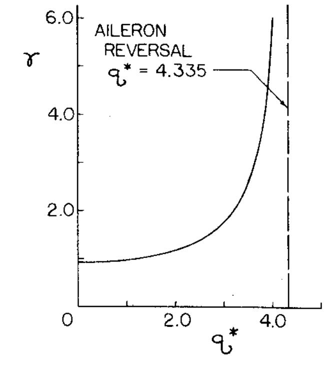

Fig. 3 The behavior of the nondimensional aileron parameter γ vs q^*.

$c_{L\alpha}$ =two-dimensional sectional lift curve slope per unit chordwise angle of attack

$c_{L\delta}$ =two-dimensional sectional lift curve slope per unit aileron deflection

If this wing were to be clamped at its center, a roll moment would be generated if ailerons were not applied in an antisymmetrical manner. This roll moment occurs because of the well-known tendency of sweptforward wings to develop lift forces when deflected upward and the tendency of sweptback wings to lose lift when similarly deflected upward. In the wind tunnel, a wing roll moment can be counteracted by the tunnel mount; in free flight, some mode of lateral control must be used. The analysis which follows assumes that longitudinal trim is supplied by control surfaces which do not enter this problem.

As shown in Ref. 7, the full span ailerons may be used to cancel the aeroelastic roll moment on the oblique wing if they are deflected such that the following relation is satisfied.

$$\delta(\eta) = \begin{cases} \delta_o & 0 \leq \eta < 1 \\ -\delta_o & -1 \leq \eta < 0 \end{cases} \tag{2}$$

Moment equilibrium about the roll axis requires that

$$\delta_o = (c_{L\alpha}/c_{L\delta})\tan\Lambda \int_{-1}^{1} (\mathrm{d}w/\mathrm{d}\eta)\mathrm{d}\eta \tag{3}$$

The existence of a very stiff wing pivot support structure at the aircraft centerline allows one to separate Eq. (1) into two regions, thus simplifying its solution. In this case, each wing half is assumed to be clamped at the aircraft centerline, or wing root position, and free of bending moments and shear at the wingtips. With these assumptions, a closed-form solution for δ_o in Eq. (3) may be found. Because the term $\mathrm{d}w/\mathrm{d}\eta$ enters into the aeroelastic load computation, this derivative is of more significance than the deflection $w(\eta)$ itself. For this reason, the solution to Eq. (1) is usually presented in terms of the variable $\Gamma(\eta)$ where

$$\Gamma(\eta) = \mathrm{d}w/\mathrm{d}\eta \tag{4}$$

With the definition of $\Gamma(\eta)$ in Eq. (4), the solution for $\Gamma(\eta)$ is written symbolically as

$$\Gamma(\eta) = \begin{cases} \Gamma_R(\eta) & 0 < \eta \leq 1 \\ \Gamma_L(\eta) & -1 \leq \eta < 0 \end{cases} \tag{5}$$

The functions $\Gamma_R(\eta)$ and $\Gamma_L(\eta)$ are given in the Appendix to this paper. Substitution of these expressions into Eq. (3) and subsequent integration yields an expression for δ_o.

$$\delta_o = \frac{p_o L^3}{\beta EI}\left(\frac{T_L - T_R}{T_L + T_R}\right) \tag{6}$$

The variables T_L and T_R are functions of the aeroelastic parameter λ and are also to be found in the Appendix.

It is found that, if the sweptforward wing portion is clamped at the root and in the absence of aileron application, the wing will undergo static divergence at a value of $\lambda = 6.33$ (Ref. 8 pp. 311-317). The value of the parameter λ thus provides one measure of the influence of static aeroelasticity in this type of problem. A related parameter which is sometimes useful is the variable q^*. The parameter q^* denotes the ratio of the flight q to the divergence q, q_{DIV}

$$q^* = q/q_{DIV} \tag{7}$$

If the variables Λ, EI, L and c are held fixed, then

$$q^* = \lambda/6.33 \tag{8}$$

The magnitude of the aileron deflection parameter δ_o may be conveniently examined if we look at the behavior of another parameter, γ, found from a manipulation of Eq. (6).

$$\gamma = 6.33\frac{\delta_o/(p_o L^3/EI)}{c_{L\alpha}\tan\Lambda/c_{L\delta}} \tag{9}$$

A graph of γ vs q^* is given in Fig. 3. Two features of the relation shown in Fig. 3 are worthy of note. First of all, as q^* approaches the value 4.335, the parameter γ tends to an infinite value because, at this value of q^*, $T_L = -T_R$. Also, it is seen that the value of γ is nearly equal to unity for values of q^* in the range $0 \leq q^* \leq 1.50$. If this latter observation is used, together with Eq. (9), then

$$\delta_o = \gamma(p_o L^3/EI)(c_{L\alpha}/c_{L\delta})\tan\Lambda/6.33 \tag{10a}$$

or

$$\delta_o \simeq (p_o L^3/EI)(c_{L\alpha}/c_{L\delta})\tan\Lambda/6.33 \tag{10b}$$

Eq. (10b) can be rendered more meaningful if p_o is related to the 1-g flight condition of an aircraft of gross weight W. p_o represents the load per unit length (assumed constant) along the swept span caused by the airloads on the rigid wing; a relation between p_o and W is readily obtained if we ignore both the aileron induced airloads and the aeroelastically induced airloads. This relation is

$$2p_o L \simeq W \tag{11}$$

Therefore, the first coefficient in Eq. (10b) becomes

$$p_o L^3/EI \simeq WL^2/2EI \tag{12}$$

The degree to which Eq. (12) is an approximation will be discussed later. However, it should be remarked here that the inclusion of aeroelastic lift into the analysis causes the term on the left hand side of Eq. (12) to be a fraction of the term on the right. This fraction is very close to unity for reasonable values of q^*. By substitution of Eq. (12) into Eq. (10b), one obtains the result

$$\delta_o \simeq [WL^2/EI]\,[c_{L\alpha}\tan\Lambda/c_{L\delta}]/12.66\,(\text{rad}) \qquad (13)$$

The most striking feature of Eq. (13) is that δ_o does not depend on the dynamic pressure and therefore is not an explicit function of the flight speed. The parameter δ_o does, however, depend upon the wing flexibility and the sweep angle Λ. Although formulated for a 1-g flight condition, the weight W enters the equation linearly and could just as well have been written as NW for an N-g condition. This feature of Eq. (13) means that, once the aircraft is trimmed for one flight speed at a given sweep angle Λ, it is trimmed for all flight speeds at that same sweep angle and altitude. To illustrate the order of magnitude of the aileron deflection which might occur for a large transport aircraft, let us use the following data which are chosen to be typical of this class of aircraft.

$$\begin{aligned} W &= 400{,}000\ \text{lb} \\ EI &= 20.0\times10^{11}\ \text{lb-in.}^2 \\ (c_{L\alpha}/c_{L\delta}) &= 2.5 \\ L &= 1200\ \text{in.} \end{aligned}$$

For this data, we obtain, from Eq. (13)

$$\delta_o \simeq 3.26[\tan\Lambda]\ (\text{deg}) \qquad (14)$$

At 45° of sweep, a full-span asymmetrical aileron deflection of 3.26° is necessary to ensure lateral equilibrium. Although not extremely large, such a deflection might have an adverse effect on aircraft yaw trim and cruise L/D.

Turning to the second objective of this paper, the previous discussion has illustrated but one method of controlling the tendency of the oblique wing to develop a rolling moment in flight. One method of improving aerodynamic performance in aircraft has been the use of geometric angle of attack distributions or "built-in-twist." For this application, the flight structure is geometrically tailored to satisfy some performance objective.

Wind tunnel tests of "rigid" oblique wings and the analysis of such wings by methods employing accurate aerodynamic theories have shown that there is a tendency for the lift distribution to build up on the sweptback wing. This tendency causes a roll moment opposite in direction to that caused by aeroelastic effects. To cancel out this adverse situation, it has been proposed that some amount of upward wing geometric curvature or dihedral be used to alleviate this roll moment.[9]

To understand how a swept wing with a built-in deflection can develop an angle of attack with respect to the airstream, consider the situation shown in Fig. 4 (the reader is referred to a similar, more complete discussion given in Ref. 10, pp. 474-479). Since small rotations transform as vector quantities, it is seen that the angle of attack α, seen by the freestream parallel to the flight direction is, for a sweptback wing section, given by the expression

$$\alpha = \Theta\cos\Lambda - \psi\sin\Lambda \qquad (15)$$

For a sweptforward wing, the preceding relation is modified by substitution of a "plus" sign for the negative sign before the second term on the right-hand side.

If Θ, the twist angle along the swept axis, is zero then the inclination of a sweptback wing such that the swept axis forms an angle ψ with the horizontal plane, i.e., a dihedral, results in a constant negative angle of attack along the wing. Conversely, a dihedral on a sweptforward wing results in a constant positive angle of attack. Thus, for an oblique wing, a

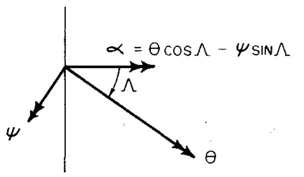

Fig. 4 Vector diagram of streamwise angle of attack caused by rotations along the swept axis.

built-in dihedral angle tends to generate a set of lift forces which roll the aircraft so as to elevate the sweptforward wing tip; negative dihedral would produce just the opposite effect. The key point here is that the dihedral effect is, in essence, a built-in geometric twist effect. The purpose of the ensuing discussion is to show how this effect can provide lateral equilibrium without any aileron action.

With the assumptions used in the previous aileron study and once again using chordwise cross sections, the nondimensional governing equation of equilibrium for the flexible wing with an initial built-in dihedral angle distribution $\psi(\eta)$ ($\psi(\eta)$ is the angle formed by the swept axis and the horizontal plane) is as follows

$$(d^4w/d\eta^4) + (\lambda\, dw/d\eta) = (p_o L^3/EI) - \lambda\psi(\eta) \quad -1\le\eta\le 1 \qquad (16)$$

The definitions of the quantities other than $\psi(\eta)$ used in Eq. (16) are identical to those used in Eq. (1). To cancel the aeroelastic roll moment, a constant initial built-in anhedral angle is used such that $\psi(\eta)$ is defined as

$$\psi(\eta) = \begin{cases} +\psi_o & -1\le\eta<0 \\ -\psi_o & 0<\eta\le 1 \end{cases} \qquad (17)$$

The comparison of Eqs. (16) and (17) with Eqs. (1) and (2) shows that they are made identical if

$$\psi_o = \delta_o\,(c_{L\delta}/c_{L\alpha})/\tan\Lambda \qquad (18)$$

To guarantee roll moment equilibrium, ψ_o must satisfy the relation

$$\psi_o = \int_{-1}^{1}\eta\,(dw/d\eta)\,d\eta = \int_{-1}^{1}\eta\Gamma(\eta)\,d\eta \qquad (19)$$

The solution for the variable $\Gamma(\eta) = (dw/d\eta)$ in this problem is identical to that presented for the previous aileron problem if the term $\lambda\psi_o$ replaces the term $\beta\delta_o$ in the expression for Γ_R and Γ_L given in the Appendix. Substitution of these expressions into Eq. (19) and subsequent integration yields a relation for ψ_o given by

$$\psi_o = \frac{1}{\lambda}\left[\frac{p_o L^3}{EI}\right]\left[\frac{T_L - T_R}{T_L + T_R}\right] \qquad (20)$$

The expressions for T_L and T_R are those presented in the Appendix.

Assumptions similar to those given for the aileron analysis lead us to an approximation for Eq. (20 given by

$$\psi_o = [WL^2/EI]/12.66 \qquad (21)$$

An examination of Eq. (21) and comparison of this relation with Eq. (13) reveals how remarkably efficient the use of built-in twist, in the form of the initial negative dihedral, is for the oblique wing roll equilibrium problem. Equation (21) has no factor $(c_{L\alpha}/c_{L\delta})\tan\Lambda$, reflecting the fact that the same airfoil sections which are causing the roll equilibrium problem are also being used for its solution. As the influence of aeroelasticity increases with the angle of sweep, so too does the counteracting built-in twist effect, since the sine of Λ in Eq. (15) increases with Λ.

From Eq. (21), it is seen that the negative dihedral angle of the swept axis is not a function of flight speed. Use of the same typical parameters as were considered for the aileron example results in a value for ψ_o of

$$\psi_o = 1.30 \text{ deg} \tag{22}$$

This initial negative dihedral angle is small and corresponds to a situation where the wing tips are initially located a distance of 27.2 in. below a horizontal plane passing through the fuselage centerline, a rather small distance when compared to the 100-ft semispan.

Since the simple formulas given by Eqs. (13) and (21) are approximations, even for the idealized model used in this analysis, two additional tasks remain before this study can be considered complete. The first of these tasks is the determination of the extent to which the aeroelastic lift contribution affects the accuracy of these strip theory expressions. The second task is to determine to what extent the use of aerodynamic strip theory to describe the loads affects the problem solution. For the sake of brevity, we will examine only the geometric twist problem, since the solutions to this problem and to the aileron problem are mathematically similar.

To accomplish the first task, we begin by summing all of the vertical forces which act upon the idealized wing and then equate these forces to the aircraft gross weight; the following expression results.

$$2\frac{p_o L^3}{EI} - \lambda \int_{-1}^{1} \mathrm{d}w/\mathrm{d}\eta \, \mathrm{d}\eta = WL^2/EI \tag{23}$$

The integral term in Eq. (23) represents the relative vertical distance between the wing tips. Physical reasoning leads one to expect that this will be a small negative quantity. It is this latter term which is ignored in the formulation of Eqs. (12) and (20).

Direct integration of the integral term in Eq. (23) using the expressions for $\Gamma(\eta)$ and subsequent solution for p_o yields the relation

$$\frac{p_o L^3}{EI} = \frac{WL^2}{2EI}\left[\frac{T_L + T_R}{T_R U_L - T_L U_R}\right] \tag{24}$$

U_L and U_R are defined in the Appendix. The substitution of Eq. (24) into Eq. (20) gives the exact solution for ψ_o

$$\psi_o = \frac{1}{2\lambda}\left[\frac{T_L - T_R}{T_R U_L - T_L U_R}\right]\left[\frac{WL^2}{EI}\right] \tag{25}$$

The coefficient of the factor $[WL^2/EI]$ is a function of the aeroelastic parameter λ. Equation (21) approximates the value of this coefficient to be 1/12.66 or 0.07900. This approximation is the result of taking the value of γ to be unity and ignoring the aeroelastic lift. The ratio $\psi_o/(WL^2/EI)$ is shown in Table 1 for several values of q^*. Comparison of the exact values of this ratio to the approximate value shows that agreement to within nearly 5% is obtained in the range $0 \leq q^* < 2.50$. Although γ begins to deviate significantly from unity for values of q^* greater than 1.50, the ratio $2p_oL^3/EI$ begins to grow smaller at the same time, as is also shown in Table 1. This reflects the fact that aeroelastic lift is becoming a larger fraction of the gross weight W.

Table 1 A comparison between approximate and exact dihedral predictions using strip theory

q^*	$\psi_o/(WL^2/EI)$ (Exact value)[a]	Percent error in approximate value	$(p_oL^3/EI)/(WL^2/2EI)$ (Exact value)
0.00	0.0750	+5.33	1.00
0.25	0.0751	+5.19	0.998
0.50	0.0753	+4.91	0.991
0.75	0.0756	+4.50	0.980
1.00	0.0761	+3.81	0.964
1.50	0.0776	+1.81	0.916
2.00	0.0797	−0.878	0.846
2.50	0.0827	−4.77	0.749
3.00	0.0867	−8.88	0.618
3.50	0.0921	−14.2	0.444
4.00	0.0994	−20.5	0.207

[a] The approximate value of $\psi_o(WL^2/EI)$ is 0.0790.

An additional feature of this exact solution is worthy of note, since it relates to the stresses in the wing structure. As the aeroelastic parameter λ increases, the proportion of the aircraft gross weight carried by each wing changes. It is found that the total lift on the sweptforward wing in the roll trimmed flight condition is given by

$$\text{S.F. Lift} = W\left[\frac{T_R U_L}{T_R U_L - T_L U_R}\right] \tag{26}$$

It follows that the portion of lift carried by the sweptback wing is equal to the weight W minus the quantity in Eq. (26). Figure 5 shows that the sweptback wing carries more of the total load as λ increases. On the other hand, the spanwise center of the lift on the sweptback wing travels inboard as λ increases. This effect of an increasing load and decreasing moment arm tends to keep changes in the wing root bending moment relatively small for reasonable values of λ.

Turning to the final topic of this paper, it has been noted that, while simple models are useful for qualitatively determining behavioral characteristics of aircraft configurations, they may be misleading in that they may overestimate or underestimate the magnitude of the phenomena. Among the possible shortcomings of the previous analysis model is the fact that strip theory aerodynamics is used; thus, aerodynamic interaction between wing segments is precluded. Even more important is the realization that the use of strip theory results in the overestimation of loads towards the wing tips.

To investigate these possible shortcomings, an analysis, based upon the theoretical model developed in Ref. 11, was carried out. The aerodynamic theory used in Ref. 11 is based upon a modification of the Weissinger-L method detailed in Ref. 12. The structural model assumes the wing to be a beam with bending-torsion flexibility. The analysis method detailed in Ref. 12 is a matrix method which permits spanwise variations in wing elastic properties.

To test the validity of the simple relations derived previously in this paper, a computer study using the model just described was conducted on an elastic wing model similar in size to a set of elastic wings used in wind-tunnel tests at Virginia Polytechnic Institute (Ref. 13). The idealized model has the following properties: $c = 4$ in.; $L = 20$ in.; $c_{L\alpha} = 6.28$ per rad; and $EI = 1.000$ lb-in.^{2} The torsional stiffness input to the computer program was chosen to be ten times the bending stiffness so that only bending flexibility was important to the analysis. The results of the analysis of this idealized, uncambered wing give the value of the initial built-in negative dihedral in terms of the wing loading parameter,

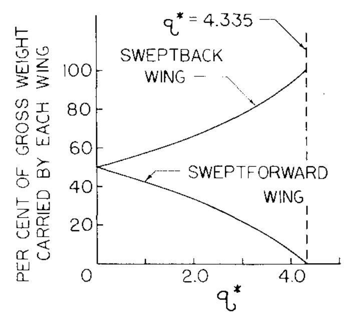

Fig. 5 Strip theory prediction of percent of gross weight W carried by each oblique wing semispan.

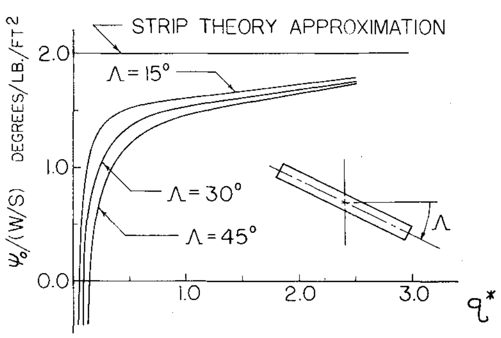

Fig. 6 Required initial anhedral (per unit W/S) vs q^* for an idealized wing.

W/S lb/ft^2. Given the scale of the idealized wing model, a value of W/S of the order of 1 to 2 psf is probably representative when compared with the large transport data given previously. For this small model, the use of Eq. (21) gives a value of

$$\psi_o = 2.012 \times (W/S) \text{deg} \tag{27}$$

The above mentioned computer analysis was run at Mach number of zero and at three sweep angles, $\Lambda = 15°$, 30° and 45°. The studies were run for q^* values in the range $0 \leq q \leq 2.5$. Attention is called to the fact that q_{DIV} changes with Λ. The results of this study are displayed in Fig. 6.

In Fig. 6, the required negative dihedral angle (per unit W/S psf.) is plotted vs q^* for the three sweep angles considered. A solid straight line is drawn to represent the strip theory prediction. From this figure, it is seen that at each value of Λ, the required value of ψ_o is a function of q^*. The change in required ψ_o is dramatic in the range $0 \leq q^* \leq 0.50$, but less so above $q^* = 0.50$. In all cases the strip theory formula overestimates the required value of ψ_o; the reason for this overestimation is that the use of strip theory results in an idealization that is too flexible from a static aeroelastic standpoint. That this latter observation is true can be seen in the fact that strip theory underestimates the value of q_{DIV} by about 30% when compared to the numerical method being used here.

Of further interest is the observation that, at low values of q^*, a negative value of ψ_o is required. This corresponds to the observation in Ref. 9 that some slight upward curvature of the wing is necessary to correct a small roll moment caused by the lift distribution shifting toward the downstream wing tip. These small values of q^* correspond to what the author would term a "rigid" wing.

Some potential importance may be attached to the fact that crossover points exist in the curves presented in Fig. 6 where the required anhedral changes sign. It is to be noted that these changes in sign occur at progressively larger values of q^* as Λ increases. At this point, by definition, no lateral trim is required to maintain equilibrium. This occurs because the roll moment caused by the airload buildup towards the sweptback wing tip is just cancelled by the aeroelastically induced load buildup towards the sweptforward wing tip. On an actual aircraft, this crossover or zero point would be a function of such parameters as wing planform shape, sweep angle, Mach number and stiffness distribution.

At the crossover point, no built-in dihedral or aileron action is necessary, no matter what the wing loading or load factor. The desirability of designing the wing so that the zero point occurs as closely to the cruise speed as possible, if not precisely at this speed, is obvious. Since so many other design objectives must be met by the structural engineer, this latter objective may be difficult to fulfill in practice.

Conclusions

Some unique static aeroelastic problems posed by the asymmetrical sweeping of a high aspect ratio wing have been examined through the use of a simplified model.

The use of strip theory to represent aerodynamic and aeroelastic loads leads to answers which overestimate the amount of aileron input or geometric twist necessary to ensure lateral equilibrium, particulary at low values of q^*. In addition, the errors introduced by strip theory give conservative results as they do when used in conventional static aeroelastic stability studies.

The results derived through the use of more accurate aerodynamic theory show that as the parameter q^* increases, increased demand is made on the system which guarantees lateral equilibrium. The parameter q^* can be made small, given a constant operating q, by increasing the clamped divergence q of the aircraft by one of several conventional methods; these methods include stiffening the structure or redistributing the wing area so that more of the area is inboard. For aircraft which are designed in a conventional manner to the the usual strength and stiffness criteria, the amount of modification to preclude the roll problem discussed in this paper is probably minimal.

Of potential interest is the use of structural modifications to further improve static aeroelastic performance. Modifications such as asymmetrical wing stiffening or redistribution of wing stiffness to bring the crossover or zero point near to the cruise q might prove to be worthwhile. A similar study of the use of various wing planforms and their relative merits is also in order.

To summarize, this problem of asymmetrical wing static aeroelastic equilibrium is one which must certainly be considered by the designers of such an aircraft. It is likely, however, that after all the conventional design criteria are met, this additional unique criteria will cause few, if any, additional problems for this aircraft configuration.

Appendix

The analytical expressions for $\Gamma_L(\eta)$, $\Gamma_R(\eta)$, $T_L(\lambda)$, $T_R(\lambda)$, $U_L(\lambda)$ and $U_R(\lambda)$ used in the body of this paper are presented below. In the region $-1 \leq \eta \leq 0$, $\Gamma(\eta)$ is given by

$$\Gamma_L(\eta) = \frac{1}{a^3}\left[\frac{p_o L^3}{EI} - \beta\delta_o\right] \times \left[1 - \frac{e^{-a(1+\eta)} + 2e^{a(1+\eta)/2}\cos f(1+\eta)}{e^{-a} + 2e^{a/2}\cos f}\right] \tag{A1}$$

where $a = \beta^{1/3}$ and $f = a(3^{1/2}/2)$.

In the region $0 \leq \eta \leq 1$, $\Gamma(\eta)$ is given by

$$\Gamma_R(\eta) = \frac{1}{a^3}\left[\frac{p_o L^3}{EI} + \beta\delta_o\right] \times \left[1 - \frac{e^{a(1-\eta)} + 2e^{-a(1-\eta)/2}\cos f(1-\eta)}{e^{a} + 2e^{-a/2}\cos f}\right] \tag{A2}$$

The expressions for T_L and T_R are given by

$$T_L = \frac{e^{-3a/2} - \cos f + 3^{1/2}\sin f}{a^2(e^{-3a/2} + 2\cos f)} \tag{A3}$$

$$T_R = \frac{e^{3a/2} - \cos f - 3^{1/2}\sin f}{a^2(e^{3a/2} + 2\cos f)} \tag{A4}$$

The expressions for U_L and U_R are

$$U_L = \frac{\cos f + 3^{1/2}\sin f - e^{-3a/2}}{a(e^{-3a/2} + 2\cos f)} \tag{A5}$$

$$U_R = \frac{\cos f - e^{1/2}\sin f - e^{3a/2}}{a(e^{3a/2} + 2\cos f)} \tag{A6}$$

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